APPLICATION OF COOPERATIVE GAME THEORY IN SUPPLY CHAIN MANAGEMENT: A SURVEY

Nikesh Kumar Srivastava, ME (ISE)
Electronic Commerce Laboratory,
Dept. of Computer Science and Automation,
Indian Institute of Science, Bangalore, India.
Email: nikesh@csa.iisc.ernet.in

Abstract

Recent emphasis on competition and cooperation on supply chain has resulted in the resurgence of game theory as a relevant tool for analyzing such situation in supply chain. This paper presents a review of papers concerned with cooperative game theoretical application in supply chain management (SCM). We first give a brief summary of cooperative game such as Shapley value, nucleous, and the core. Our review of application of cooperative game theory is presented in following areas: (1) Inventory Games, (2) Production and pricing competition. We will provide the means for supply chain partners selection for supply chain formation based on supply chain optimization problem (MVA). We will show that when payment is made by using Shapley value the resulted coalition is stable and no manager will find worth to deviate from coalition and truth telling is best strategy for each stage manager. The paper concludes with summary of our review and suggestion for potential use of game theory in SCM.

Index Terms

Supply chain management, cooperative game, coalition, cooperation, the core, Shapley value, optimization, mean variance allocation, rational, supply chain formation, supply chain planner (central design authority).

Acronyms

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I. INTRODUCTION

Game theory is concerned with the analysis of situation involving conflict and cooperation. Since its development in the early 1940s game theory has found many application in divers areas such as auction, business, economics, politics and philosophy. After the initial excitement generated by it potential applications, interest in game theory by operations research/management science specialist seemed to waned during 1960s and 1970s. However, the last two decades have witnessed a renewed interest by academics and practitioners in the management of supply chains and new emphasis on interaction among the decision makers constituting a supply chain. This has resulted in the proliferation of publications in scattered journal dealing with the use of game theory in analysis of supply chain related problems. The purpose of this paper is to provide a wide ranging survey focusing on cooperative game theoretic applications in different area of supply chain management.

A supply chain can be define as a system of suppliers, manufactures, distributors, retailers and customers where material flow downstream from suppliers to customers and inormation flows in both directions Ganeshan et al. [60]. Supply chain management, on the other hand, is defined by some researcher as set of management processes. For example Lalonde [6] define SCM as ”the process of managing relationship, information, and material flow across enterprise borders to delivery enhanced customer service and economics value through synchronized management of flow of physical goods and associated information from sourcing to consumption” (see Mentzer et al [30] for collection of competing definitions.) Adopting LaLonde’s definition, one observes that the most SCM-related research has features that are common to operations management and marketing problems.

A. Contribution and Outline of the Report

After scanning plethora of the survey paper we found that all the survey paper describe both cooperative and non-cooperative game as solution concept for the SCM problems. There are very few papers which investigate the cooperative game theory concepts such as, the core, Shapley value, and Nucleolus as a solution concept for the SCM problems. This fact motivate us to come up with a survey paper which investigate the cooperative game theory concepts as a solution approach for the SCM problems such as, cooperative inventory and price and production cooperation among the players. Further more we have addressed a very common problem known as supply chain partners selection for the supply chain formation which result in high level of delivery performance say (Six-Sigma) while ensuring that each agent is telling truthful to the supply chain planner (central design authority). To ensure this we have came with a supply chain optimization problem which known as mean variance allocation problem which provides a mean for the selection of the best mix of the partners among numbers of alternative service provider. The money (payment) to each stage manger is made by using shapley value approach which result in very stable coalitions and each stage manger found that even in coalition it is good for them to reveal their true private information. To support our idea we have formulated a three echelon linear supply chain as a case study. The rest of the paper is organized as follows:

- In section II, we see related work in this area.
- In section III, we define cooperative game and provide brief description of cooperative game theory solution concepts such as, the core, Shapley value, and the nucleolus.
- In section IV, we provide a list of the papers which investigated the cooperative game theory to solve the inventory problems in SCM.
- In section V, we provide the list of paper which investigated the cooperative game theory to solve production and pricing competition in SCM.
- In section VI, we address supply chain formation problem in general.
- In section VII, we provide the Shapley value approach for supply chain formation for the n echelon linear supply chain with stochastic lead time.
In section VIII, we describe a three echelon linear supply chain to support our notion in previous section.
In section IX, we summarize the contribution of the paper.

II. RELATED WORK

Several survey paper related to SCM have appeared in literature.
1) Tayur et al. [72] have edited a book emphasizing quantitative models for SCM.
2) Ganeshan et al. [60] proposed a taxonomic review and frame work that help both practitioners and academic researchers better understand up-to-date state of SCM research.
3) Wilcox et al. [27] presented a brief survey of the papers on price-quantity discount.
4) McAlister [34] reviewed a model of distribution channels incorporating behavior dimensions.
5) Goyal and Gupta [68] provided a survey of literature that treated buyer-vendor coordination with integrated inventory models.

In addition to above, some reviews focusing on the application of game theory in economics and management science have appeared in last five decades.

- An early survey of game theoretic applications in management science is given by Shubik [43].
- Feichting and Jorgensen [16] published a review that was restricted to application of differential game in management science and operation research.
- Li and Whang [37] provided a survey of game theoretic model applied in operations management and information systems where the SCM-related literature focusing on information sharing and manufacturing/marketing incentives was also discussed.
- Mingming and Mahmut [35] have provided a wide-ranging survey of more than 130 papers focusing on game theoretic applications in different areas of supply chain management.
- Nagarajan and Sosic [45] have provided a survey paper on some application of cooperative game theory in supply chain management. They have given special emphasis on two aspects of cooperative games: profit allocation and stability.
- Cachon [19] reviewed the literature on supply chain collaboration with contracts.
- Cachon and Netessine [21] outlined game theoretic concepts and surveyed applications of game theory in supply chain management. Cachon and Netessine classified games that were developed for SCM into four categories based on game theoretical techniques: (i) Non-cooperative static games, (ii) dynamics games, (iii) cooperative games, and (iv) signaling, screening and Bayesian games. In each category, the authors presented the major techniques that are commonly used in existing papers and those that could be applied in future research.
- Srivastava and Narahari [49] have provided a incentive compatible method of selecting supply chain partners to achieve high level of delivery performance in supply chain based on VCG and DAGVA mechanism [1], [11].
- In additions several books Chatterjee and Samuleson [31], Gautschi [10], Kuhn and Szego [25] and Sheth et al. [29] partially reviewed some specific game related topics in SCM.

To get the taxonomy of game theory in supply chain management (SCM) see figure 1

III. COOPERATIVE GAME THEORY

A game in characteristic function form, or simply a game, is a pair \((N, v)\), where \(N\) is a finite set (the set of players), and \(v\) is a real-valued function on the family of subsets of \(N\) with \(v(\emptyset) = 0\). The function \(v\) itself will also be called a game, or a game on \(N\). The set of all games on \(N\) is denoted \(G^N\); \(G^N\) is an Euclidean space of dimension \(2^N - 1\), where \(|N|\) is the cardinality of \(N\).

A payoff vector for \(N\) is a real-valued function \(x\) on \(N\); it may be thought of as a vector whose coordinates are indexed by the players. If \(S \subset N\), write \(x(S) = \sum_{i \in S} x(i)\). The set of all payoff vectors
for $N$ is denoted $E^N$. It is sometimes useful to constrain the set of payoff vectors under consideration to a subset $X$ of $E^N$; we therefore define a constrained game \([17], [65]\) to be a triple $(N, v, X)$, where $(N, v)$ is Cooperative Games with Coalition Structures a game and $X \subset E^N$. When there is no constraint, then $X = E^N$; thus $(N, v)$ may be identified with $(N, v, E^N)$. We will use the term ”game” for a constrained game as well; no confusion should result.

A coalition structure $\beta$ on $N$ is a partition of $N$, the generic element of which will be denoted $B_k$. A game with coalition structure $\beta$ is a triple $(N, v, \beta)$ The analysis of $(N, v, \beta)$ differs from that of $(N, v)$ in two respects:

1) Payoff vectors associated with $(N, v, \beta)$ usually satisfy the conditions $x(B_k) = v(B_k)$ for all $k$ (no side-payments between coalitions); in particular, these conditions are imposed by all the solution concepts considered below.

2) In addition, the partition $N$ enters directly into the definition of certain of the solution concepts (namely, the value, the bargaining set and the kernel). The conditions stated in 1 may easily be replaced by constraints on the set of payoff vectors. Given a game $(N, v)$, define:

$$X_\beta = \{ x \in E^N : x(B_k) = v(B_k) \forall k, \quad x_i \geq v(\{i\}) \forall i \}.$$  

As will be seen below, the games $(N, v, \beta)$ and $(N, v, X_\beta)$ are equivalent from the point of view of some, but not all, solution concepts. We also find it convenient to define

$$X_k = \{ X \in E^{\beta_k} : X(B_k) = v(B_k) \quad x_i \geq 0 \quad \forall i \in B_k \}.$$  

A O-normalized game is a game for which $v(i) = 0$ \(\forall i\). If $(N, v)$ is a 0- normalized game, then $X_\beta = \times_k X_k$. In general, however, there is a distinction between the definition of $X_\beta$, which includes the conditions $x \geq v(i)$, and the definition Of $X_k$, which includes the conditions $x_i \geq 0$.

A permutation $\pi$ of $N$ is a one-one function from $N$ onto itself. For $S \subset N$, write $\pi S = \{ \pi i : i \in S \}$. If $v$ is a game on $N$, define a game $\pi_* v$ on $N$ by

$$(\pi_* v)(S) = v(\pi S).$$
A. Shapley Value

Fix $N$ and $\beta$. A $N$-value is a function $\phi_\beta$ from $G^N$ to $E^N$ -i.e. a function that associates with each game a payoff vector -obeying the following conditions:

- Relative efficiency:
  \[ \forall k, (\phi_\beta)(B_k) = v(B_k). \] (6)

- Symmetry: For all permutations $n$ of $N$ under which $N$ is invariant
  \[ (\phi_\beta(\pi_*v))(S) = (\phi_\beta v)(\pi S). \] (7)

- Additivity:
  \[ \phi_\beta(v + w) = \phi_\beta v + \phi_\beta w. \] (8)

- Null-Player condition: If $i$ is a null-player, then
  \[ (\phi_\beta v)(i) = 0. \] (9)

When $\beta = \{N\}$, it is known that there is a unique function $\phi_\beta$ satisfying through 5-8, namely the usual SHAPLEY value of the game [67]; it will be denoted by $\phi$. This notation will be maintained even for games whose player set differs from $N$; thus if $v$ is a game with player set $M$, $\phi v$ is defined to be $\phi_\beta v$, where $\beta = \{M\}$.

For each $S \subset N$, denote by $v|S$ the game on $S$ defined for all $T \subset S$ by
\[ (v|S)(T) = v(T). \]

**Theorem 1:** Fix $N$ and $\beta = (B1..., Bp)$. Then there is a unique $\beta$ and it is given $\forall k = 1, ... , p$, and all $i \in B_k$, by
\[ (\phi_\beta v)(i) = (\phi(v|B_k))(i). \] (10)

**Remark:** (9) asserts that the restriction to $B_k$ of the value $\phi_\beta$ for the game $(N, v)$ is the value $\phi$ for the game $(B_k, v|B_k)$. In other words, the value of a game with coalition structure $\beta$ has the "restriction property": The restriction of the value is the value of the restriction of the game. An important implication of this property is that $\phi_\beta$ can be computed by computing separately $\phi(v|B_k)$ for each $k$. Proof of the Theorem 1 can be seen in [61].
B. The Nucleus

Let \((N, v, X)\) be a constrained game. For each \(x \in X\), let \(\theta(x)\) be a vector in \(E^{2n}\), the elements of which are the excesses \(e(x, S)\) for \(S \subset N\), arranged in order of non-increasing magnitude; i.e. \(\theta_s(x) \geq \theta_t(x)\) whenever \(t > s\). Write \(\theta(y) \geq \theta(x)\) (or \(\theta(y) > \theta(x)\)) if and only if \(\theta(x)\) is not greater (or is smaller) than \(\theta(y)\) in the lexicographic order on \(E^{2n}\). The nucleolus, w.r.t., the set \(X\), is then defined by

\[
Nu(N, v, X) = \{x \in X : \theta(y) \geq \theta(x) \quad \forall y \in X\}.
\]  

(11)

For a coalition structure \(\beta\), we define \(Nu(N, v, \beta) = Nu(N, v, X_\beta)\). In particular, when \(\beta = \{N\}\), we write \(Nu(N, v) = Nu(N, v, \{N\})\).

When \(X \neq 0\), the nucleolus consists of a single element [33], [66]; this element, as well as the set of which it is the only member, will also be called the nucleolus. Thus, like the value and unlike other solution concepts, the nucleolus assigns to each game precisely one payoff vector.

We saw that the restriction to \(B_k\) of the value for \((N, v, \beta)\) is the value for \((B_k, v|B_k)\). Does a similar property hold for the nucleolus? The answer is no.

**Theorem 2:** Let \((N, v)\) be a O-normalized game, and let \(x = Nu(N, v, \beta)\). Then \(Nu(N, v, N)|Bk = Nu(B_k, v|B_k, X_k)\).

**Corollary 1:** Let \((N, v)\) be a O-normalized game, decomposable with partition \(\beta = (B1, ..., Bp)\). Then \(Nu(N, v, \beta) = \times_{i=1}^p Nu(B_k, v|B_k, X_k)\).

Remark:

A similar result holds for the SHAPLEY value; but in that case, it holds for all games, not only decomposable games.

Proof of the Theorem 2 and Corollary 1 be seen in [61].

C. The Core

The core of the game \((N, v, X)\) is defined by

\[
Co(N, v, X) = \{x \in X : e(x, S) < 0 \forall S \subset N\}.
\]  

(12)

For a coalition structure \(N\), we define \(Co(N, v, N) = Co(N, v, X_\beta)\). In particular, when \(\beta = \{N\}\), we write \(Co(N, v) = Co(N, v, \{N\})\).

The core does not have the uniqueness property of the nucleolus. Accordingly, it could not have the ”restriction property” of the value. But one could raise questions such as the following:

1) Does \(x \in Co(N, v, \beta)\) imply \(x|B_k \in Co(B_k, v|B_k)\) ?

2) Does \(y \in CO(B_k, v|B_k)\) imply \(y = x|B_k for some x \in Co(N, v, \beta)\)?

The answer to question (1) is positive, but the answer to question (2) is negative.

**Theorem 3:** Let \((N, v)\) be a O-normalized game, and let \(x \in Co(N, v, \beta)\). Then the section of \(Co(N, v, \beta)\) at \(x|\)—\(N\backslash B_k\) is \(Co(B_k, v_x^*, X_k)\).

Proof of the theorem 3 can be seen in [61].

**Corollary 2:** Let \((N, v)\) be a O-normalized game, decomposable with partition \(\beta = (B1,...,Bp)\). Then \(Co(N, v, \beta) = \times_{i=1}^p Co(B_k, X_k)\).

This result was proved by MASCHLER, PELEG, and SHAPLEY [40], as Lemma. It also follows from Theorem 3 and decomposability upon noting that, when \(x \in Co(N, v, \beta)\), \(v_x^*(S) = v(S) + max_{T \subset N \backslash B_k} v(T) - x(T) = v(S)\).

IV. INVENTORY GAME

Inventory management problems involving competition arise either in horizontal and vertical channels. First, we consider the competition in horizontal channels. We try to review all the scattered literature which involve cooperative game as solution concept for the problems.
• Parlar [42] developed a context game theoretic model of competition between two players. In his model the product sold by the retailers are substitutable and retailers simultaneously choose their order quantity to maximize their expected profits. The retailer’s profit is function of demand densities and substitution rate when they are sold out. For this model Parlar proved existence and uniqueness of Nash equilibrium and showed that cooperation between two retailers can increase their profits.

• Wang and Parlar [56] extended the model to describe a three-person game in same context as described above. They also investigated the cooperation of retailers when switching excess inventory between three players (side-payment) is not allowed. They showed that the Nash equilibrium exists in both the cases and cooperation reduces inventory.

• Recently Avsar and Baykal-Gursoy [85] extended Parlar’s model in [42] to infinite horizon and lost-sales case and examined a two-person nonzero-sum stochastic game under the discounted payoff criterion.

• Lippman and McCardle [63] analyzed a competitive newsboy model in both oligopoly and duopoly contexts. They started duopoly case with two aspects of demand allocation: the initial allocation and reallocation. With initial allocation, they specified several rules to split demands to various firms. The reallocation is same cooperative scheme (side-payment) as in Wang and parlar [56].

• Anupindi, Bassok, and Zemel [59] developed a general framework for the analysis of two stage distributed systems where \( N \) retailers face stochastic demands. More specifically, in the first (non-cooperative) stage each retailers decide on his order quantity to satisfy his own demand. In second Stage, the retailers tranship product for the residual demands and allocate the corresponding additional profits. The authors derived conditions for existence of a Nash equilibrium in first stage, and in second stage used the concept of core for the allocation of profit and also provided sufficient conditions for the existence of core.

• Garnot and Sosic [13] extended the result in Anupindi et al. [59] to a three-stage are the same as the first and second stages [59], and in second each retailer decides how much of his residual he wants to share with other retailers.

A few papers have also been published emphasizing cooperative inventory system.

• Gerchak and Gupta [82] examined the allocation of joint inventory control costs among multiple (\( N \)) customers of a single supplier. They first proved that centralization always beneficial in this model. The authors also showed that the control cost for the model have the super additive feature.

• As a extension of Gerchak and Gupta’s work Robinson [38] showed that best allocation approaches in preceding work is unstable i.e., not in the core of associated game. Robinson also pointed out that the Shapley value as allocation scheme satisfies stability.

• Hartman and Dror [3] re-examined the cost allocation scheme for the centralized and continuous-review inventory system. In their work three criteria (stability, justifiability and polynomial computability) are proposed to evaluate seven allocation methods including Shapley value discussed in Robinson [38] and nucleolus scheme.

• Meca et al. [2] develop a simple inventory model with \( n \) retailers of an identical product who face deterministic, constant demand. The model assumes zero lead time, and that the retailers incur order placing cost and holding cost. The firms can cooperate to reduce their ordering costs. The authors assume that firms share only information on their individual optimal annual number of orders, and develop a proportional rule, to allocate joint ordering cost. This cost only depends on the ordering cost and the individual average number of orders per time unit, which is public information. An interesting property of this allocation rule is that it results in same decisions as if information about players individual costs and demands has been revealed.

• Reinhardt and Dada [22] in a brief note consider a model with \( n \) firms who cooperate by pooling their critical resources. The benefits generated through this cooperation are distributed among the firms according to Shapley value. As the Shapley value can be difficult to compute with a large number of players, the authors develop an algorithm that computes it in pseudo-polynomial time for
a particular class of games, which they call coalition symmetric.

- Kemahlıoğlu Ziya [83] analyzes a supply chain consisting of one supplier and n retailers facing stochastic independent demand. The supplier can keep inventory reserved for each retailer, or form a coalition with the retailers and pool the inventory to share among them. In the latter case, the total profit is distributed among players according to Shapley value. The author shows that Shapley allocations coordinate the supply chain and are individually rational.

- Rudi, Kapur and Pyke [48] investigated a two-location inventory problem with transhipment.

- Hartman et al. [23] prove the non-emptiness of the core for the single period inventory model with n players facing demands with symmetric distributions and for player facing joint multivariate normal demand distribution.

- Hartman and Dror [24] show the non-emptiness of the core for single period inventory model with n players facing normally distributed, correlated individual demands.

- Muller et al. [44] strengthen their result by showing non-emptiness of the core for all possible joint distribution of demand. They also provided condition under which the core is singleton.

- Slikker et al. [70] enrich this model by allowing retailer to use transhipment (at positive cost) after demand realization is known. They show that core is non-empty even if the retailers have different retail and wholesale price.

- Chen and Zhang [81] formulate the inventory centralization problem as a stochastic linear programming (LP), and show that non-emptiness of the core directly from strong duality of stochastic LP. In addition, the stochastic approach provides a way to compute an element in the core. The authors also show that non-emptiness of the core for the news-vendor game with more general, concave ordering cost. Finally, they show that determining whether an allocation is in core of the news-vendor game is NP hard.

- Hartman and Dror [5] study model of inventory centralization for n retailers facing random correlated demands. They consider two different games- one based on expected cost (benefits), and the other based on demand realization. For the first case, they show the non-emptiness of the core when holding and shortage cost are identical for all subset of retailers, and demand is normally distributed. However, the core can be empty when retailers holding and penalty differ. For the second case, the core can be empty even when retailers are identical.

- Hartman and Dror [4] consider a model with joint ordering in which the cost of ordering an item has two separable components - a fixed cost independent of item type, and an item specific cost. The authors address two questions - what items should be ordered together, and how should the ordering costs be allocated among the players. They show that the core of the game is non-empty when items should be ordered together. However, if the shared portion of the ordering cost is too small, the core may be an empty set.

- Klijn and Slikker [15] study a location-inventory problem with m customers and n distribution centers (DCs). Demand at each demand point is modeled as a continuous stochastic process, and it is assumed that demands are identically and independently distributed. DCs follow a continuous review policy with a positive fixed leadtime, and all stockouts are backordered. The inventory costs consist of ordering and holding cost. DCs may cooperate by forming a coalition and reassigning the initial demand within the coalition to minimize costs. It is assumed that the customers are indifferent about where their orders are shipped from, and that the outbound transportation costs do not depend on where the orders are shipped from. Under these conditions, the authors show non-emptiness of the core.

- Ozen et al. [77] consider a game with n news-vendors, m warehouses, and a supplier, in which the retailers are supplied from the warehouses. The retailers can increase their expected joint profits by coordinating their initial orders and redistributing the ordered quantities at the warehouse, after demand realization. They show that this game has a non-empty core.

- Ozen and Sosic [75] extend this model by assuming that reallocation of inventories happens after a demand signal is observed. The signal updates the information about the demand distribution, but may
not reveal the true demand realization. The authors analyze the impact of two contracting schemes (the wholesale price contract and the buy-back contract) in three models (non-cooperating retailers, cooperating retailers, and manufacturers resale of returned items) on the manufacturers profit, and study the conditions for achieving a system-optimal solution. They show that the core is not empty under both contracting schemes.

- In a previously mentioned paper, Leng and Parlar [41] show the non-emptiness of the core show the non-emptiness of the core for their information-sharing game.

- Ben-Zvi [46] and Ben-Zvi and Gerchak [47] study news-vendor games in which the retailers that have different shortage costs centralize their inventories. They analyze several approaches to the distribution of stocks in the case of a shortage, and show that these games have a non-empty core.

- Ozen et al. [76] analyze the convexity of some simple newsvendor games. As a result of the convexity, the marginal vectors of a game are the extreme points of the core, the bargaining set coincides with the core, and the Shapley value is the barycenter of the core. While the news-vendor games are not convex in general, the authors concentrate on the class of news-vendor games with independent symmetric unimodal demand distributions and identify cases that lead to the convex games.

Now we restrict our attention to the vertical competition issues related to inventory control.

- Cachon and Zipkin [20] investigated a two-stage supply chain with stationary stochastic demand and fixed transportation time. The authors provided two different game under two tracking methods for firm specified as supplier and retailer. In a competitive setting either game has a unique Nash equilibrium. Under conditions of cooperation with simple linear transfer payments (side-payment) it was also claimed that global supply chain optimal solution can be archived as a Nash equilibrium.

- Cachon [18] also extended the above model to analyze the competitive and cooperative inventory issue in a two echelon supply chain with one supplier and $N$ retailers.

- Raghnathan [71] considered a one manufacturer, $N$-retailer supply chain with correlated demand at retailers and applied the Shapley value concept to analyze the expected manufacturer and retailer share of the surplus incurred due to information sharing and relative incentives of manufacturer and retailer to form information sharing partnerships.

- Corbett [8] who studied the well known $(Q, r)$ model in supplier-buyer supply chain with conflicting objective and asymmetric information.

- Keen reader can also read the papers by Anupindi and Bassok [58], Anupindi and Bassok [57] and Axsater [64].

**INVENTORY GAME WITH QUANTITY DISCOUNT**

Quantity discount policy is common marketing scheme adopted in many industries. With this scheme buyer has an incentive to increase his/her purchase quantity to obtain a lower unit price. In recent years several reviews focusing on quantity discount have been published including Chiang et al. [79] and Wilcox et al. [26].Since quantity discount plays an important role in the analysis of two-stage vertical supply chains.

- Monahan [28] developed and analyzed a quantity discount model to determine the optimal quantity discount schedule for a vendor. The paper considered the scenario in which vendor or buyer are involved in Stackelberg game model. Monahan assumed that vendor requests the buyer to increase his/her order size by a factor $K$ and performed the analysis to determine buyer response. As one of the early work on quantity discount decisions, Monahan [28] is an important contribution to literature. However, Joglekar [51] pointed out some shortcoming of [26] as well as its contribution.

- Lal and Stalín investigated the same problem [62] respectively under the cooperative and competitive environment.

- Extending Lal and stalin’s work Kohli and Park [7] examined a cooperative game theory model of quantity discount to analyze a transaction-efficiency rational for quantity discount offered in
bargaining context. In this model, a buyer and seller negotiate over lot size orders and the average unit price. The authors used the Pareto optimal approach to investigate the Pareto efficient transactions.

- Kim and Hawang [32] studied the effect of the quantity discount on supplier’s profit and buyer’s cost in cooperative and competitive context. They explored how the supplier decides the discount schedule given the assumption that the buyer always behaves optimally by using classical EOQ inventory decision.
- Chiang et al. [79] investigated game theoretic discount problem in two-stage competition and cooperative context. For the cooperative game the Pareto optimal criterion was utilized to find multiple optimal strategies. They concluded that quantity discount is a mechanism of coordinating channel members.
- Weng [84] presented a model for analyzing the impact of joint decision policies on channel coordination in a supply chain including a supplier and group of homogenous buyers.
- Li and Huang [53] addressed the problem regarding cooperation between seller and buyer.
- Chen, Federgruen and Zheng [14] adopted a power-of-two policy to coordinate the replenishment within a decentralized supply chain with one supplier and multiple retailer.
- Wang [54] considered a similar decentralized supply chain and developed a coordination strategy that combine integer-ratio time and uniform quantity discounts. Wang showed that the integer-ratio time provides a better coordination mechanism the the power-of-two coordination used in [14].

V. PRODUCTION AND PRICING COMPETITION

Some of the earliest applications of game theoretical ideas were in production and pricing competition and they can be tracked back to the 19th century. Since production and pricing decisions play an important role in the profitable operations of supply chain we now review some paper in this topics.

- The first publication emphasizing the channel cooperation in this category was by Zusman and Etgar [52] with the combined application of economics contract theory and Nash bargaining theory. Individual contract involving payment schedules between members of a three level channel were investigated and equilibrium contract obtained.
- McGuire and Staelin [74], four industry structure induced by two type of channel system consisting of two manufacturer were studied. Under the assumption that one seller (retailer) carries the product of only one manufacturer, they derived the Nash equilibrium price, quantities and profit for each of four different structures. An extension of the cooperative game model in [74] was again proposed by McGuire and Staelin [73] by extending non-cooperative model in [74]
- In recent publication Li [36] has examined the incentive for firm to share information vertically for improving the performance of a single manufacturer, N retailer supply chain. In the supply chain, the retailer are engaged in Cournot competition and manufacturer determine the wholesale price. The condition under which information can be shared were derived in the paper.
- Larivire [39] considered supply chain coordination issues with random demand under several contract schemes such as price-only contracts, buyback contract and quantity-flexibility contracts.
- Corbett and DeCroix [9] developed shared-shaving contract for indirect material in supply chain containing supplier and buyer (customer).

VI. THE SUPPLY CHAIN FORMATION PROBLEM

Supply chains provide the backbone for manufacturing, service, and e-business companies. The supply chain is a complex, composite business process comprising a hierarchy of different levels of value-delivering business processes. In today’s global supply chain environment, very few firms have complete control over the entire supply chain. Even in the case where firm owns significant portion of the entire supply chain, different segments are likely to be owned by different organization. This because in recent years, the business world has realized that business efficiency can be improved and risk can be minimized if the firm just focus on building their core competencies and outproduce the peripheral business processes.
Today companies no longer take ownership of all the assets and processes, that are required to form a supply chain for delivering value to customer. Instead they focus on their core competencies and partner with companies possessing complementary strength. This puts the companies in better competitive position.

This paradigm shift in business strategy has given rise to the formation of supply chain networks and emergence of intermediate such as third-party logistic providers. Contrast manufactures and electronics marketplaces in almost all industries. Thus, a modern supply chain network can be viewed as a collection of independent companies, possessing complementary skills and integrated with streamlined material, information, and financial flows that work together to meet market demand. Many of these network are controlled by original equipment manufacturers(OEMs) or channel master, who own the brand of new product and select other manufacturing and logistics partners in supply chain networks based on characteristic such as the requirement of the partner, and the total cost of order fulfillment. It is no longer enough to be merely the best-of-bread manufacturer or contract manufacturer. It is also critical to partner with best-of-breed companies for other supply chain function such as component manufacturing, logistic, maintains, testing, etc [50].

In view of this scenario, selecting the appropriate partner for each supply chain stage which is not owned by firm itself, is an extremely important strategic decision making problem for industrial supply chains. At an abstract level, we can define the supply chain formation problem as the problem of determining the best mix of partners for each supply chain stage which is not owned by the firm itself [80]. This problem occurs more frequently in industries where the product life cycle is very short and underlying supply chain is dynamics in nature which requires its configuration to be changed more frequently. The IT supply chain is very good example of dynamics supply chain. The manufacturing firm also faces the similar problem of supply chain formation but less frequently than IT firms. The supply chain formation problem has many avatars depending upon the nature of the industry, business goals and objective, and constrains such as market demand, delivery time, budget, etc. In this chapter we will focusing on a class of supply chain planner (or CDA) is to choose the partner for each supply chain stage that ensure delivery targets and schedules to be met at minimum.

VII. SUPPLY CHAIN FORMATION SHAPLEY VALUE APPROACH

Let us consider an $n$-echelon linear supply chain with stochastic lead times. See Figure 2, which shows a make-to-order linear supply chain consisting of $n$ stages. Let us assume the following.

1) For each stage of supply chain, there exist multiple partners in the market who can offer the service required at that stage.

2) At each stage, the time taken to deliver one unit of the order by each partner is stochastic in nature. $X_i$ represents the delivery time for one unit of order at stage $i$. $X_i$ is normally distributed with mean $\mu_i$ and standard deviation $\sigma_i$.

3) A potential supply chain partner for stage $i$ may offer different values for the mean $\mu_i$, standard deviation $\sigma_i$ and delivery cost of one unit of order $\nu_i$. The delivery cost $\nu_i$ typically depends on $\mu_i$ and $\sigma_i$.

4) The manager for each stage $i$ seeks, from each partner, information such as cost of each partner and corresponding values of $\mu_i$ and $\sigma_i$ offered by the partner. The partners provide this information, not necessarily truthfully. For the moment, we assume that they provide the cost curves in a truthful way. Later on in the paper, we relax this assumption and investigate the incomplete information case. The managers use this information to come up with an aggregated cost function $\nu_i(\mu_i, \sigma_i)$ for stage $i$. This aggregate cost function can be obtained, for example, by performing a polynomial curve fitting over the available quotations from the various partners for stage $i$. Such an aggregate cost function $\nu_i(\mu_i, \sigma_i)$ will capture the cost versus delivery performance trade-offs across different partners available for stage $i$.

5) The delivery time $X_i$ of the various stages in the $n$-echelon supply chain are independent random variables and there is no time elapsed between end of process $i$ and commencement
of process \( i + 1 \) \( \forall i = 1, \ldots, n - 1 \). As the consequence, the end-to-end delivery time \( Y \) of an order is

\[
Y = \sum_{i=1}^{n} X_i
\]

It can noted immediately that \( Y \) is normally distributed with \( \mu = \sum_{i=1}^{n} \mu_i \) and variance \( \sigma^2 = \sum_{i=1}^{n} \sigma_i^2 \).

---

**Fig. 2.** A linear multi echelon supply chain with stochastic lead time

1) **Characterizing the Delivery Performance in terms of Supply Chain Process Capability Indices:** We assume that the CDA's target is to deliver the orders to the respective customers within \( \tau \pm T \) days of receiving the order. We call \( \tau \) as the target delivery time and \( T \) as tolerance. We also define \( L = \tau - T \) to be the lower limit of the delivery window and \( U = \tau - T \) be the upper limit of the delivery window. The CDA measures the delivery performance of the supply chain in terms of how precisely and accurately the order are being delivered to the customer within the delivery window \( (\tau \pm T) \). We can say that the delivery performance of the supply chain depends on the the mean \( \mu \) and standard deviation \( \sigma \) of the end-to-end delivery time \( Y \). Because mean \( \mu \) and standard deviation depends on the values of mean \( \mu_i \) and standard deviation \( \sigma_i \) of the delivery time for each stage of the supply chain, there may be a number of ways in which the CDA can choose the partner for each stage of the supply chain so that the corresponding set of parameter \( (\mu_i, \sigma_i)_{i=1,\ldots,n} \) will result in mean \( \mu \) and standard deviation \( \sigma \) which would render the desired level of delivery performance. However, there might be some way of choosing the partner for each stage of the supply chain so that end-to-end delivery cost gets minimized together with achieving the desired level of delivery performance. The objective of the CDA is to form such a supply chain. Thus, we can see that the problem of the CDA is to minimize end-to-end delivery cost subject to delivery performance constraints. We formulate this optimization problem in terms of the supply chain process capability indices \( C_p, C_{pk} \) and \( C_{pm} \), which are popular in the area of design tolerating and statistical process control [12], [69], [78].
The CDA first invites each stage manager to submit its cost function \( \nu_i(\mu_i, \sigma_i) \). For the moment, assume that each stage manager is loyal to CDA and submits true cost function to CDA. After receiving the true cost function from each stage, the CDA just needs to solve an optimization problem that will minimize the expected end-to-end delivery cost while ensuring the specified levels of delivery performance. The solution of the optimization problem results in optimal values of design parameter \( (\mu_i^*, \sigma_i^*) \) which are communicated back to the respective stage managers (see Figure 3) by the CDA. The CDA also allocates budget \( \kappa_i = \nu_i(\mu_i, \sigma_i) \) for the manager of stage \( i \) \( \forall i = 1, 2, ..., n \).

![Diagram of the centralized design paradigm](image)

**Fig. 3.** The idea behind centralized design paradigm

**Known Parameters:** The optimization problem faced by the CDA can be formally defined in form of known parameters, decision variables, objective function, and constraints. The following parameters are known to the CDA.

1) The delivery window \((\tau, T)\)
2) Lower bounds on the values of \( C_p \) and \( C_{pk} \), say for example \( C_p \geq p \) and \( C_{pk} \geq q \).
3) Delivery cost function \( \nu_i(\mu_i, \sigma_i) \) per unit order submitted by the manager of stage \( i \).

**Decision Variables:** The decision variables are optimal means \( \mu_i^* \) and optimal standard deviations \( \sigma_i^* \) of each individual stage \( i \) \( (i = 1, ..., n) \).

**Objective Function and Constraints:** As stated already, the objective function is to minimize the end-to-end delivery cost and the constraints are specified in terms of minimum expected level of \( C_p \) and \( C_{pk} \) on end-to-end delivery time. Thus, the problem formulation becomes:
Minimize $\sum_{i=1}^{n} \nu_i(\mu_i, \sigma_i)$ \hspace{1cm} (13)

Subject to:

$C_p \geq p$ \hspace{1cm} (14)

$C_{pk} \geq q$ \hspace{1cm} (15)

$\tau - T \leq \sum_{1}^{n} \mu_i \leq \tau + T$ \hspace{1cm} (16)

$\mu_i \geq 0, \sigma_i \geq 0 \hspace{1cm} \forall i.$ \hspace{1cm} (17)

An interesting approach to the above optimization problem is to solve it as a *Mean Variance Allocation* (MVA) problem. The cost function depends on $\sigma_i$ as well as $\mu_i$. We assume that the cost function $\nu_i(\mu_i, \sigma_i)$ has the following form

$$\nu_i(\mu_i, \sigma_i) = a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2 + a_{i5}\mu_i\sigma_i^2$$ \hspace{1cm} (18)

The function $\nu_i(\mu_i, \sigma_i)$ is a polynomial which varies linearly with mean $\mu_i$ and quadratically with standard deviation $\sigma_i$. The manager of each stage submits a 6-tuple $(a_{i0}, a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5})$ to the CDA. Thus, the problem of the CDA is to decide optimal standard deviation $\sigma_i^*$ and mean $\mu_i^*$ for each stage $i$ so as to achieve the delivery performance goals.

### A. Shapley value approach for payment to each stage manager

Payment to each stage manager can be made by using Shapley Value. The payment to each stage manager by CDA is given by:

$$I_i = \sum_{S \subseteq N} \frac{(|S| - 1)! (N - |S|)!}{N!} [\nu(S) - \nu(S - \{i\})]$$ \hspace{1cm} (19)

Payment scheme needs to satisfy the following:

$$\sum_{S \subseteq N} I_i \geq \nu(S)$$ \hspace{1cm} (20)

and

$$\sum_{i \in N} I_i = \nu(N) = \alpha$$ \hspace{1cm} (21)

Where $\alpha$ is total budget of the CDA.
To illustrate the implication of Shapley value approach we consider the problem of forming a three echelon supply chain network as shown in Figure 4. We assume that the manager is autonomous, rational, and intelligent and hence may not report the true cost function to the CDA. The objective behind the current experiment is to show that even if each stage manager are in fully cooperative environment and payment is made by Shapley value enable the CDA to elicit the true cost curves from each manager. We make the following specific assumptions in this case study.

1) The CDA manager has an ideal target of finishing the process on \((30 \pm 5)\text{th}.\) days

2) There are three stages in supply chain say, casting stage, machining stage, and transportation stage.

3) The first stage has 4 service providers available, the second stage has 5 service providers available, and third stage has 6 service providers available.

4) For each stage, the variability, mean delivery time, and the cost vary across alternate service providers. This is private information of the manager for that stage and his information is known neither to the CDA nor to the remaining two managers. Tables I, II, and III show the private information available with the individual manager that we will use in our study. Note that we are assuming that the costs at each stage depends on mean delivery time and variability.

5) The delivery time at each leg is normally distributed for all the service providers. The delivery times at the three stages are mutually independent.

6) The CDA wants to choose the service provider for each stage in a way that the values \(C_p^* \geq 1.8\) and \(C_{pk}^* \geq 1.08\) are attained for the end-to-end cycle time \(Y\) which is equal to the sum of the delivery times of the three stages.

### VIII. A Case Study of a Three Echelon Linear Supply Chain

To Illustrate the implication of Shapley value approach we consider the problem of forming a three echelon supply chain network as shown in Figure 4. We assume that the manager is autonomous, rational, and intelligent and hence may not report the true cost function to the CDA. The objective behind the current experiment is to show that even if each stage manager are in fully cooperative environment and payment is made by Shapley value enable the CDA to elicit the true cost curves from each manager. We make the following specific assumptions in this case study.

1) The CDA manager has an ideal target of finishing the process on \((30 \pm 5)\text{th}.\) days

2) There are three stages in supply chain say, casting stage, machining stage, and transportation stage.

3) The first stage has 4 service providers available, the second stage has 5 service providers available, and third stage has 6 service providers available.

4) For each stage, the variability, mean delivery time, and the cost vary across alternate service providers. This is private information of the manager for that stage and his information is known neither to the CDA nor to the remaining two managers. Tables I, II, and III show the private information available with the individual manager that we will use in our study. Note that we are assuming that the costs at each stage depends on mean delivery time and variability.

5) The delivery time at each leg is normally distributed for all the service providers. The delivery times at the three stages are mutually independent.

6) The CDA wants to choose the service provider for each stage in a way that the values \(C_p^* \geq 1.8\) and \(C_{pk}^* \geq 1.08\) are attained for the end-to-end cycle time \(Y\) which is equal to the sum of the delivery times of the three stages.

### Table I

<table>
<thead>
<tr>
<th>Partner Id</th>
<th>(\mu) (days)</th>
<th>(\sigma) (days)</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_{11})</td>
<td>3</td>
<td>2.5</td>
<td>105</td>
</tr>
<tr>
<td>(P_{12})</td>
<td>3</td>
<td>3.0</td>
<td>70</td>
</tr>
<tr>
<td>(P_{13})</td>
<td>2</td>
<td>3.5</td>
<td>55</td>
</tr>
<tr>
<td>(P_{14})</td>
<td>2</td>
<td>4.0</td>
<td>45</td>
</tr>
</tbody>
</table>
7) The manager for each stage $i (i = 1, 2, 3)$ uses his/her private information to compute the true cost function

$$
\nu_i(\mu_i, \sigma_i) = a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2 + a_{i5}\mu_i\sigma_i^2
$$

using the polynomial curve fitting method reflecting linear variation with mean and quadratic variation with standard deviation, to compute the coefficients $a_{i0}, a_{i1}, a_{i2}, a_{i3}, a_{i4}$ and $a_{i5}$.

8) Total budget $\alpha = 600$.

**A. Non-Cooperative Environment when Each Stage Manager are Truthful**

For the sake of completeness we will start from non-cooperative environment where each stage manager are independently dealing with the CDA. In this case it is just an optimization problem (MVA) and they will paid according to the solution of the optimization problem. The value of coefficients is shown in Table IV. For the above case $\text{s}$ The CDA needs to solve the following optimization problem.

Minimize:

$$
\kappa = \sum_{i=1}^{3} (a_{i0} + a_{i1}\mu_i + a_{i2}\sigma_i + a_{i3}\mu_i\sigma_i + a_{i4}\sigma_i^2 + a_{i5}\mu_i\sigma_i^2)
$$

subject to

$$
\sigma_i^2 + \sigma_2^2 + \sigma_3^2 \leq \frac{T^2}{9C_p^2} = \frac{d^2}{9C_{pk}^2} \leq \frac{25}{29.16}
$$

$$
25 \leq \mu_1 + \mu_2 + \mu_3 \leq 35
$$

$$
\sigma_i \geq 0 \quad \forall i = 1, 2, 3
$$

$$
\mu_i \geq 0 \quad \forall i = 1, 2, 3.
$$

Hence solving the objective function described above by using the Lagrange multiplier method will yield the values shown in Table V.
### TABLE IV

**THE VALUE OF COEFFICIENTS OF EACH STAGE MANAGER BASED ON THEIR TRUTHFUL PRIVATE INFORMATION**

<table>
<thead>
<tr>
<th>Stage i</th>
<th>$\mu_i^*$ days</th>
<th>$\sigma_i^*$ days</th>
<th>$\nu_i(\mu_i^<em>, \sigma_i^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.9131</td>
<td>384.8845</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.1431</td>
<td>73.9732</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>0.0</td>
<td>5.6</td>
</tr>
</tbody>
</table>

### TABLE V

**OPTIMAL VALUES OF DECISION VARIABLES AND OPTIMAL COST OF EACH STAGE MANAGER WHEN THEY ARE TRUTHFUL**

#### B. Fully cooperative Environment when Each Stage Manager are Truthful

The coalition containing stage 1 are $S = \{1\}, \{12\}, \{13\},$ and $\{123\}$:

$$S = \{1\}; |S| = 1 \implies \frac{(1-1)!(3-1)!}{3!} = \frac{1}{3}$$

$$(27)$$

$$S = \{12\}, \{13\}; |S| = 2 \implies \frac{(2-1)!(3-2)!}{3!} = \frac{1}{6}$$

$$(28)$$

$$S = \{123\}; |S| = 3 \implies \frac{(3-1)!(3-3)!}{3!} = \frac{1}{3}$$

$$(29)$$

The coalition containing stage 2 are $S = \{2\}, \{12\}, \{23\},$ and $\{123\}$:

$$S = \{2\}; |S| = 1 \implies \frac{(1-1)!(3-1)!}{3!} = \frac{1}{3}$$

$$(30)$$

$$S = \{12\}, \{23\}; |S| = 2 \implies \frac{(2-1)!(3-2)!}{3!} = \frac{1}{6}$$

$$(31)$$

$$S = \{123\}; |S| = 3 \implies \frac{(3-1)!(3-3)!}{3!} = \frac{1}{3}$$

$$(32)$$

The coalition containing stage 3 are $S = \{3\}, \{13\}, \{23\},$ and $\{123\}$:

$$S = \{3\}; |S| = 1 \implies \frac{(1-1)!(3-1)!}{3!} = \frac{1}{3}$$

$$(33)$$

$$S = \{13\}, \{23\}; |S| = 2 \implies \frac{(2-1)!(3-2)!}{3!} = \frac{1}{6}$$

$$(34)$$

$$S = \{123\}; |S| = 3 \implies \frac{(3-1)!(3-3)!}{3!} = \frac{1}{3}$$

$$(35)$$

The payment to first stage manager by the CDA:

$$I_1 = \frac{1}{3}[\nu(1) - \nu(0)] + \frac{1}{6}[\nu(12) - \nu(2)] + \frac{1}{6}[\nu(13) - \nu(3)] + \frac{1}{3}[\nu(123) - \nu(23)]$$

$$(36)$$

$$I_1 = \frac{1}{3}[384.8845] + \frac{1}{6}[384.8845] + \frac{1}{6}[384.8845] + \frac{1}{3}[(600 - (73.9732 + 5.6))] = 430.0646$$

$$(37)$$

The payment to second stage manager by the CDA:

$$I_2 = \frac{1}{3}[\nu(2) - \nu(0)] + \frac{1}{6}[\nu(12) - \nu(1)] + \frac{1}{6}[\nu(23) - \nu(3)] + \frac{1}{3}[\nu(123) - \nu(13)]$$

$$(38)$$
The payment to third stage manager by the CDA described above by using the Lagrange multiplier method will yield the values shown in Table VII.

For this, we assume that the managers submit untruthful values as shown in Table VI.

We compute the budget based on Shapley value then they are able to get more money from the CDA. Hence Equation (42)- (48) is total money the CDA is suppose to distribute across stage managers. But the purpose of the CDA is to minimize the money. When each stage manager agree for the cooperation and allocation rule is made sure coalition is stable and allocated payment belongs to core.

Here, the constraints will be the same as in Equations (23) - (26). Hence solving the objective function described above by using the Lagrange multiplier method will yield the values shown in Table VII. The

\[
I_2 = \frac{1}{3}[73.9732] + \frac{1}{6}[73.9732] + \frac{1}{6}[73.9732] + \frac{1}{3}[(600 - (384.8845 + 5.6))] = 119.1485
\]  

The payment to third stage manger by the CDA:

\[
I_3 = \frac{1}{3}[\nu(3) - \nu(0)] + \frac{1}{6}[\nu(13) - \nu(1)] + \frac{1}{6}[\nu(23) - \nu(2)] + \frac{1}{3}[\nu(123) - \nu(12)]
\]

\[
I_3 = \frac{1}{3}[5.6] + \frac{1}{6}[5.6] + \frac{1}{6}[5.6] + \frac{1}{3}[(600 - (73.9732 + 384.8845))] = 50.7927
\]

\[
I_1 = 430.0646 > 384.8845 = \nu_1(\mu_1^*, \sigma_1^*) = \nu(1)
\]

\[
I_2 = 119.1485 > 73.9732 = \nu_2(\mu_2^*, \sigma_2^*) = \nu(2)
\]

\[
I_3 = 50.7927 > 5.6 = \nu_3(\mu_3^*, \sigma_3^*) = \nu(3)
\]

\[
I_1 + I_2 + I_3 = 549.2131 > 458.8577 = \nu_1(\mu_1^*, \sigma_1^*) + \nu_2(\mu_2^*, \sigma_2^*) = \nu(12)
\]

\[
I_2 + I_3 = 169.9412 > 79.5372 = \nu_2(\mu_2^*, \sigma_2^*) + \nu_3(\mu_3^*, \sigma_3^*) = \nu(23)
\]

\[
I_1 + I_3 = 480.8573 > 390.4845 = \nu_1(\mu_1^*, \sigma_1^*) + \nu_3(\mu_3^*, \sigma_3^*) = \nu(13)
\]

\[
I_1 + I_2 + I_3 = 600 = \alpha = \nu(123)
\]

Here \(\alpha\) is total money the CDA is suppose to distribute across stage managers. But the purpose of the CDA is to minimize the money. When each stage manger agree for the cooperation and allocation rule is based on Shapley value then they are able to get more money from the CDA. Hence Equation (42)- (48) make sure coalition is stable and allocated payment belongs to core.

Now for the purpose of convincing the mangers that reporting the true cost function is good for them, we compute the budget \(I_i\) to the manager in the case when each of them reveals the untruthful cost function. For this, we assume that the managers submit untruthful values as shown in Table VI.

It is easy to verify that \(w_i(\mu_i, \sigma_i) \geq \nu_i(\mu_i, \sigma_i)\) \(\forall(\mu_i, \sigma_i); \forall i = 1, 2, 3\). In such a situation CDA solves the following optimization problem: Minimize:

\[
\kappa = \sum_{i=1}^{3} (a_{i0} + a_{i1} \mu_i + a_{i2} \sigma_i + a_{i3} \mu_i \sigma_i + a_{i4} \sigma_i^2 + a_{i5} \mu_i \sigma_i^2)
\]

Here, the constraints will be the same as in Equations (23) - (26). Hence solving the objective function described above by using the Lagrange multiplier method will yield the values shown in Table VII.
payment to first stage manager by the CDA:

\[ I_1 = \frac{1}{3}[w(1) - w(0)] + \frac{1}{6}[w(12) - w(2)] + \frac{1}{6}[w(13) - w(3)] + \frac{1}{3}[w(123) - w(23)] \]  

(50)

\[ I_1 = \frac{1}{3}[462.2995] + \frac{1}{6}[462.2995] + \frac{1}{6}[462.2995] + \frac{1}{3}[(600 - (21.0859 + 83.4629))] = 415.0345 \]  

(51)

The payment to second stage manager by the CDA:

\[ I_2 = \frac{1}{3}[w(2) - w(0)] + \frac{1}{6}[w(12) - w(1)] + \frac{1}{6}[w(23) - w(3)] + \frac{1}{3}[w(123) - w(13)] \]  

(52)

\[ I_2 = \frac{1}{3}[83.4629] + \frac{1}{6}[83.4629] + \frac{1}{6}[83.4629] + \frac{1}{3}[(600 - (462.2995 + 21.0859))] = 94.51 \]  

(53)

The payment to third stage manager by the CDA:

\[ I_3 = \frac{1}{3}[w(3) - w(0)] + \frac{1}{6}[w(13) - w(1)] + \frac{1}{6}[w(23) - w(2)] + \frac{1}{3}[w(123) - w(12)] \]  

(54)

\[ I_3 = \frac{1}{3}[21.0859] + \frac{1}{6}[21.0859] + \frac{1}{6}[21.0859] + \frac{1}{3}[(600 - (462.2995 + 83.4629))] = 32.148 \]  

(55)

By equations (50)-(55) we can show that even if each stage manager are in fully cooperative environment truth telling is good for them.

IX. Summary

The objective of supply chain is to maximize the overall values (profitability, deliver performance, customer satisfaction, etc). The supply chain profitability is the total profit distributed across all supply stage; Naturally, such profitability can be achieved only if the decision makers in each stage of each stage of supply chain agree to cooperate. For supply chain researchers interested in applying game theory should be an exciting observation. We have here basic ingredients of a cooperative game, i.e., a group of decision maker having different objective; and if they cooperate they can improve their well-being as a whole. Since our survey reveled that very few research have looked at problem in SCM involving cooperative game in characteristic function form one may develop a cooperative game of a supply chain involving the major ”players” of supply chain, i.e., the supplier, manufacturer, distributor, retailer and customer. The problem of sharing fairly the increased profit in a supply chain may be analyzed by using the solution concept of game theory such as Shapley value or nucleolus.

In particular we have shown in our case study when the payment is made by using Shapley value the coalition formed by the each stage manger is very stable. The payment scheme describe by using Shapley value is turn out to be an incentive for each stage manger to reveal their private information truthfully while, working in fully cooperative environment.
REFERENCES
